USN | | | | | | | | | | **13MCA12**

First Semester MCA Degree Examination, June / July 2014
Discrete Mathematical Structures

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. 2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

1 a. For any three non-empty sets A, B and C prove that,
i) $(A-B)-C = A-(B\cup C) = (A-C)-(B-C)$
ii) $(A\cap(B-C)) = (A\cap B)-C$ (using laws of sets) **(06 Marks)**

b. Among the integers 1 – 200, determine the number of integers that are,
i) divisible by 2 or 5 or 9.
ii) not divisible by 2 or 5 or 9.
iii) divisible by 5 or not by 2 or 9. **(07 Marks)**

c. In how many ways can one distribute eight identical balls into four distinct containers so that,
i) no container is left empty?
ii) the fourth containers gets an odd number of balls. **(07 Marks)**

2 a. Define Tautology and contradiction, prove that the proposition,
$[(p\rightarrow r)\wedge(q\rightarrow r)]\rightarrow[(p\vee q)\rightarrow r]$ is a Tautology. **(07 Marks)**

b. Prove the logical equivalence using laws of logic.
i) $[(\neg p\vee\neg q)\rightarrow(p\wedge q\wedge r)]\Leftrightarrow(p\wedge q)$
ii) $(\neg p\wedge(\neg q\wedge r))\vee((q\wedge r)\vee(p\wedge r))\Leftrightarrow r$ **(06 Marks)**

c. Write the following argument in symbolic form and hence establish the validity of the statements. If Rochelle gets the superwiser's position and works hard, then she will get a raise. If she gets a raise, then she will buy a car. She has not purchased a car. Therefore either Rochelle did not gets the supervisor's position or she did not work hard. **(07 Marks)**

3 a. Define an open statement and quantifiers. Write the negation of the statement ; If k, m and n are any integers, when (k – m) and (m – n) are odd, then (k – n) is even. **(06 Marks)**

b. For the universe of all real numbers, let $p(x): x\geq 0$, $q(x): x^2\geq 0$, $r(x): x^2-3x-4=0$. Find the truth values of the following statements. If a statement is false, provide a counter example.
i) $\exists x,(p(x)\wedge q(x))$ ii) $\forall x,(p(x)\rightarrow q(x))$ iii) $\exists x,(p(x)\wedge r(x))$ **(07 Marks)**

c. Using quantifiers, find whether the following statement is valid or not.
If a triangle has two equal sides, then it is isosceles. If a triangle is isosceles, then if has two equal angles. The triangle xyz does not have 2-equal angles. Therefore the triangle xyz does not have two equal sides. **(07 Marks)**

4 a. Prove by mathematical induction, for all positive integer $n\geq 1$, that
$$1^2+3^2+5^2+.....(2n-1)^2=\frac{n(2n+1)(2n-1)}{3}$$ **(06 Marks)**

b. Apply back-tracking technique to obtain an explicit formula for the sequence defines by recurrence relation $b_n = 2b_{n-1}+1$ with the initial condition, $b_1 = 7$. **(07 Marks)**

c. For any $n\in z^+$, prove that the integers (8n+3) and (5n+2) are relatively prime. **(07 Marks)**

5 a. Prove that a function f: A → B is invertible, if and only if, it is one-to-one and on-to. **(06 Marks)**

b. Define : i) function ii) one-to-one function and iii) on-to function.
Let f : z → z be defined by f(a) = a + 1 for all a ∈ z. Find whether f is a one-to-one correspondence or not. **(07 Marks)**

c. State pigeonhole principle. Show that if any 14 integers are selected from the set s = {1, 2, 3, 4,.....25}, there all at-least two integers whose sum is 26. **(07 Marks)**

6 a. If A = {1, 2, 3, 4} and R is a relation on A defined by R = {(1, 2), (1, 3), (2, 4), (3, 2), (3, 3), (3, 4)} find M(R), $M(R^2)$ and verify that $M(R^2) = [M(R)]^2$ **(06 Marks)**

b. Define an equivalence relation with example, For a fixed integer n > 1, prove that the relation "Congruent modulo n" in an equivalence relation on the set of all integers z. **(07 Marks)**

c. Define partial order with an example. Let A = {a, b, c} and P(A) be the power set of A. On P(A), define the relation R by 'xRy' iff $x \le y$. Draw the 'Hasse-diagram' of the poset (P(A),R). **(07 Marks)**

7 a. Define the following with an example to each,
i) Connected and disconnected graph.
ii) Isomorphic graphs
iii) Complete bipartite graphs. **(06 Marks)**

b. Define Hamilton cycle. How many edge-disjoint Hamilton cycles exist in the complete graph with seven vertices? Also draw the graph to show these Hamilton cycles. **(04 Marks)**

c. How many different palls of length 2 are there in the undirected graph G in Fig. Q7 (c)? **(04 Marks)**

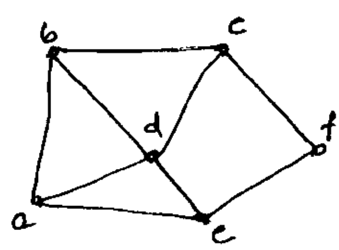

Fig. Q7(c)

d. Define chromatic number of a graph. Prove that a graph of order $n (\ge 2)$ consisting of a single circuit is 2-chromatic if n is even, and 3-chromatic if n is odd. **(06 Marks)**

8 a. Define with an example for each of the following : i) Rooted tree ii) Complete binary tree iii) Balanced tree. **(06 Marks)**

b. Construct an optimal prefix code for the symbols a, o, q, u, y, z that occur with the frequencies 20, 28, 4, 17, 12, 7 respectively. **(07 Marks)**

c. Define spanning tree with an example. Prove that a graph is connected if an only if it has a spanning tree. **(07 Marks)**

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